خطاهای اندازه گیری در تحقیق مواد

درس دوره کارشناسی ارشد مهندسی مواد

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دانشکده بین المللی امام خمینی

مطالب:
مقدمات
عملیات اعداد
مفهوم آماری
روش‌های آماری
برنامه ریزی آزمایش
روش‌های آماری
<table>
<thead>
<tr>
<th>Category</th>
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<th>Basic Example</th>
<th>Student Task</th>
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<tbody>
<tr>
<td>Dualism/ Received Knowledge</td>
<td>Basic Duality</td>
<td>All problems are solvable. The authorities know.</td>
<td>All problems are solvable. The authorities know.</td>
<td>The tutor doesn't know what is right and wrong but others do.</td>
<td>Learn the Right Solutions</td>
</tr>
<tr>
<td></td>
<td>Full Dualism</td>
<td>There are Right Solutions, the true authorities are right, the others are frauds.</td>
<td>There are Right Solutions, the true authorities are right, the others are frauds.</td>
<td>My tutor doesn't know what is right and wrong but others do.</td>
<td>Learn the Right Solutions and ignore the others.</td>
</tr>
<tr>
<td></td>
<td>Early Multiplicity</td>
<td>There are 2 kinds of problems: those whose solutions we know, those whose solutions we don't know yet.</td>
<td>There are 2 kinds of problems: those whose solutions we know, those whose solutions we don't know yet.</td>
<td>There are some uncertainties and the authorities are working to find the truth.</td>
<td>Learn the Right Solutions and ignore the others.</td>
</tr>
<tr>
<td></td>
<td>Late Multiplicity</td>
<td>Most problems are of the second kind. (a) Everyone has right to their own opinion. (b) The authorities don't want the right answers. They want us to think in certain way.</td>
<td>Most problems are of the second kind. (a) Everyone has right to their own opinion. (b) The authorities don't want the right answers. They want us to think in certain way.</td>
<td>Different nations think different things. There is no answer that the nations want and we have to find it.</td>
<td>Learn to evaluate solutions</td>
</tr>
<tr>
<td>Multiplicity/ Subjective Knowledge</td>
<td>Contextual Relativism</td>
<td>All proposed solutions are supported by reasons. Some solutions are better than others, depending on context. Everything is relative but not equally valid.</td>
<td>All proposed solutions are supported by reasons. Some solutions are better than others, depending on context. Everything is relative but not equally valid.</td>
<td>There are no right and wrong answers, it depends on the situation. But some answers might be better than others.</td>
<td>Learn to evaluate solutions</td>
</tr>
<tr>
<td></td>
<td>Pre- Commitment</td>
<td>Student sees the necessity of: a-making choices b-committing to a solution</td>
<td>Student sees the necessity of: a-making choices b-committing to a solution</td>
<td>What is important is not what the tutor thinks but what I think.</td>
<td>Make his/her own decisions</td>
</tr>
<tr>
<td></td>
<td>Commitment</td>
<td>First commitment For this particular topic I think....</td>
<td>First commitment For this particular topic I think....</td>
<td>For this particular topic I think that...</td>
<td>For this particular topic I think that...</td>
</tr>
<tr>
<td></td>
<td>Challenges to Commitment</td>
<td>Student experiences implications of commitment.</td>
<td>Student experiences implications of commitment.</td>
<td>For these topics I think that...</td>
<td>For these topics I think that...</td>
</tr>
<tr>
<td></td>
<td>Pre- Commitment</td>
<td>Student realizes commitment is an ongoing, unfolding, evolving activity.</td>
<td>Student realizes commitment is an ongoing, unfolding, evolving activity.</td>
<td>What I believe is not what the tutor thinks but what I think.</td>
<td>Believe in own values, respect others, be ready to learn.</td>
</tr>
<tr>
<td>Commitment Constructed Knowledge</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

References:

Horst Czichos, Tetsuya Saito, Leslie Smith (Eds.)
Springer Handbook of Materials Measurement Methods

Paolo Fornasini
The Uncertainty in Physical Measurements

Jiju Antony
Design of Experiments for Engineers and Scientists
Science?

Physical science?

Scientific method?

Physical quantity?

Methods of Observation and Measurement

**Morphological Method:** sensorial detection of some properties by drawings, photographs, etc.

**Classificatory Method:** partitioning a set of objects or phenomena into classes

**Comparative Method:** introducing an order relation

A property for which a comparative method can be defined is a physical quantity.

**Quantitative Method:** correspondence between the degrees of the physical property and the set of real numbers; introducing an additive structure

**Statistical Methods:** statistical methods play a fundamental role in the treatment of uncertainties of physical quantities
Exact numerical value (exact numbers)

- sin 30° = 0.5
- 1 foot ≡ 12 inches
- \( c \approx 3 \times 10^8 \text{ m.s}^{-1} \)
- \( \cos 30° \approx 0.8660254 \approx 0.866 \)

Approximate numerical values

- \( c \approx 3 \times 10^8 \text{ m.s}^{-1} \)
- \( \cos 30° \approx 0.8660254 \approx 0.866 \)
**Significant Figures**

Significant Figures \( S_f \)

Significant Digits \( S_d \)

Number of significant figures

---

The number of significant figures is the minimum number of digits needed to write a given value in scientific notation without loss of accuracy.

1236 \( 1.236 \times 10^3 \) (4 significant figures)

0.0123 \( 1.23 \times 10^{-2} \) (3 significant figures)

2700 \( 2.7 \times 10^3 \) (2 significant digits)

2700 \( 2.70 \times 10^3 \) (3 significant digits)

2700 \( 2.700 \times 10^3 \) (4 significant digits)

---

47000

The number of significant figures is the minimum number of digits needed to write a given value in scientific notation without loss of accuracy.

---

**47000** ?

---
The number 25.04 has 4 significant digits: 2 5 0 4

(Leading zeros)

The number 0.0037 has 2 significant digits: 3 7

(Tailing zeros)

The number 0.50 has 2 significant digits: 5 0

The first digit on the left: **The most significant digit** MSD

The last digit on the right: **The least significant digit** LSD

\[
\begin{align*}
X &= 2.47 \text{ m} & \text{instead } X &= (2.47 \pm 0.005) \text{ m} \\
X &= 2.470 \text{ m} & \text{instead } X &= (2.470 \pm 0.0005) \text{ m}
\end{align*}
\]

*The first uncertain digit is the last significant figure.*

\((12.30(\pm 0.01) \text{ mL})\)
Rounding off

It is best to carry extra digits through intermediate calculations and round the final answer to the correct number of significant digits.

For additions and subtractions of approximate numbers: the digits of the sum or difference are not significant to the right of the position corresponding to the leftmost of the least significant digits of the starting terms.

EXAMPLES:

\[ 83.17 \text{g} + 0.041 \text{g} = 83.211 \text{g} = 83.21 \text{g} \]
\[ 9.253 \text{g} - 1.3 \text{g} = 7.953 \text{g} = 8.0 \text{g} \]

\[ \begin{array}{c}
2.456 + \\
0.5 + \\
3.35 = \\
\hline
6.306 \rightarrow 6.3
\end{array} \]

For multiplications and divisions of approximate numbers: it is reasonable to round off the result to \( n \), or sometimes \( n + 1 \) significant digits, where \( n \) is the smallest between the numbers of significant digits of the factors.

• EXAMPLES:

\[ 1.2 \text{mol} \times 3.4231 \text{g/mol} = 4.10772 \text{g} = 4.1 \text{g} \]
\[ 0.012 \text{L} \times 0.013 \text{mol/L} = 1.56 \times 10^{-4} \text{mol} = 1.6 \times 10^{-4} \text{mol} \]
\[ 6.83 \times 72 = 491.76 = 4.9 \times 10^2 \]

\[ 83.642 \div 72 = 1.1616944 = 1.2 = 1.16 \]
The square roots of approximate numbers are generally rounded off to the same number of significant digits of the radicand.

\[
\sqrt{30.74} = 5.5443665 \text{ is rounded off to } 5.544
\]

significant digits \(\longleftrightarrow\) uncertainty

The number of significant digits of a measure is determined by the extent of its uncertainty.

Notes:

(a) The uncertainty \(\delta X\) should be expressed by no more than two significant digits, and sometimes one significant digit is sufficient.
(b) When a measurement result is expressed as \(X_0 \pm \delta X\), the least significant digit of \(X_0\) should be of the same order of magnitude as the least significant digit of the uncertainty.
mean, median, mode
(average)

The mean of a collection of numbers is found by adding the numbers together, and then dividing by however many numbers there are in the collection.

The **median** is the middle number of the collection of numbers when they are written in order, either from least to greatest or from greatest to least.

The **mode** of a collection of numbers is the most frequently occurring number (or numbers) in the collection.

**Range?**

The **range** of a set of numbers is the difference between the greatest and least numbers in the set.
display resolution $\Delta X$
measurement uncertainty $\delta X$

$Example\ 4.1$ The thickness of a metal foil is measured by a micrometer with display resolution $\Delta X = 0.01\ mm$. The measure can be influenced by the presence of dust between the micrometer rods and the foil (interaction between instrument and system). If the micrometer has been calibrated at $20^\circ C$ and is used to perform measurements at much lower or much higher temperatures, nonnegligible errors can be introduced (environment influence). The thickness of the metal foil can be different in different parts of the foil (difficulty in defining the physical quantity).

$Example\ 4.2$ The period of a pendulum is measured by a manual stopwatch with display resolution $\Delta t = 0.01\ s$. The measure will depend on the quickness of reflexes of the experimenter (interaction between instrument and experimenter). Moreover, the result can be different according to whether the duration of a single period is measured, or the duration of ten periods is measured and then divided by ten (measurement methodology).
Error

Accuracy

Precision

Typed of errors

- measurement resolution
- random fluctuations (or “random errors”)
- systematic errors
**Instrument Resolution** and **Measurement Resolution**

\[ \Delta X_{\text{meas}} = \frac{\Delta X_{\text{inst}}}{n} \]

*Example 4.4.* The period \( T \) of a pendulum is again measured by a stopwatch with display resolution \( \Delta t_{\text{inst}} = 0.01 \text{s} \). The duration \( \Delta T \) of \( n = 10 \) consecutive oscillations is measured, and the period is calculated as \( T = \Delta T/10 \). The measurement resolution of the period \( T \) is thus \( n = 10 \) times smaller than the instrument resolution: \( \Delta t_{\text{meas}} = \Delta t_{\text{inst}}/10 = 0.001 \text{s} \).

---

**Resolution interval**

\[ X_{\text{min}}, X_{\text{max}} \]

- Resolution interval
- Uniform density distribution
- \( \delta X = \Delta X/2 \)
- Maximum uncertainty
- \( \delta X = \Delta X/\sqrt{12} \)
- Standard uncertainty

\[ \delta X_{\text{max}} = \frac{\Delta X}{2} \quad \delta X_{\text{res}} = \frac{\Delta X}{\sqrt{12}} \]

*maximum resolution uncertainty* *standard resolution uncertainty*
accuracy, resolution, precision and error

Accuracy is the extent to which a measurement approaches the true value of the measured quantity.

Precision is the quality of being exactly or sharply defined or stated.

Resolution is the smallest increment that can be distinguished and acted upon.\(^{(i)}\),\(^{(ii)}\)

“Error” is inversely related to “accuracy” (and “precision”, ??) and defines how far a series of experimental result vary from the expected or programmed value.

\[
\frac{1}{\text{Accuracy}} \propto \text{Error} \propto \frac{1}{\text{Precision}}
\]


Precision: repeatability of measurement.

Accuracy: agreement between measurement and the true value.
Random errors

(a)

(b)

\[ \Delta X \]
Normal distribution

\[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{(x - \mu)^2}{2\sigma^2} \right] \]
standard deviation error limits

Examples ...
توزیع میانگین‌ها

\[ D[m^*] = \frac{1}{N} D \]

\[ \sigma[m^*] = \frac{1}{\sqrt{N}} \sigma \]

\[ \delta X_{\text{cas}} = \sigma[m^*] = \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^{N} (x_i - m^*)^2} \]

Cas = casual (random)
**Systematic error**

\[ |X_A - X_B| \leq \frac{\Delta X_A + \Delta X_B}{2} \]

consistent

\[ |X_A - X_B| > \frac{\Delta X_A + \Delta X_B}{2} \]

inconsistent
\[ X_A \pm \delta X_A, \quad X_B \pm \delta X_B \]

weights of the values \( X_A \) and \( X_B \):

\[ w_A = \frac{1}{(\delta X_A)^2}, \quad w_B = \frac{1}{(\delta X_B)^2} \]

weighted average \( X_w = \frac{X_A w_A + X_B w_B}{w_A + w_B} \)

uncertainty \( \delta X_w \) of the weighted average:

\[ \delta X_w = \frac{1}{\sqrt{w_A + w_B}} \]

\[ X_w = \frac{\sum_i X_i w_i}{\sum_i w_i}, \quad \text{where } w_i = \frac{1}{(\delta X_i)^2} \]

\[ \delta X_w = \frac{1}{\sqrt{\sum_i w_i}} \]
دیتاهاي پرت و تشخیص آنها

Outliers
(extreme values)

ریشه های ایجاد:

۱- خطاهاي خين انجام آزمایش

۲- نمونه های غير عادي یا متفاوت

\[
\delta X_{sys} \sim \frac{|X_A - X_B|}{2}
\]
روش‌های تشخیص آماری

گرافیکی

روش‌های تشخیص آماری

Strip chart

Dot plot

هیستوگرام

خطاهای انتقاله گیری در نخستین مرحله.
اصول تشخیص و حذف:

1- اگر دیتا از حد اطمینان 99% خارج باشد، دیتا حذف می‌شود.
اینکه دلیلی (غیر آماری) برای حفظ دیتا وجود داشته باشد.

2- اگر دیتا از حد اطمینان 95% خارج باشد، دیتا فقط در صورتی حذف می‌شود که دلیل دیگری (غیر آماری) برای حفظ دیتا وجود داشته باشد.

3- در هر صورت نباید حذف دیتا‌های برت منجر به حذف تعداد قابل وجوه از داده‌ها شود.

Example:

\[
\begin{align*}
&1, 2, 3, 4, 5, 6, 7 \\
&\text{Mean} = \frac{28}{7} = 4 \\
&\text{Median} = 4
\end{align*}
\]

\[
\begin{align*}
&1, 2, 3, 4, 5, 6, 60 \\
&\text{Mean} = \frac{81}{7} = 11.6 \\
&\text{Median} = 4
\end{align*}
\]
Quartiles

Example:

Range:

46, 32, 48, 36, 47, 35, 31, 27, 44, 39

\[
\text{Range} = 48 - 27 = 21
\]

\[
\text{داده های مرتب شده:}
27, 31, 32, 35, 36, 39, 44, 46, 47, 48
\]

Quartiles?

\[
\begin{array}{ccc}
\text{Lower Quartile} & \text{Upper Quartile} \\
LQ & UP \\
\end{array}
\]

\[
\text{median}
\]
Interquartile range?

\[ 46 - 32 = 14 \]

Box and whiskers method
Median ± 1.5 × interquartile range

MN ≡ [ Mean ± 2.7 S.D.] ≡ 99% confidence interval
Normal distribution

\[ P = \int_{-\delta}^{\delta} \frac{1}{\sqrt{2\pi}\sigma} \exp \left( -\frac{\varepsilon^2}{2\sigma^2} \right) d\varepsilon \]

Student t distribution

\textbf{N<20?}

\textit{student t distribution}

shape depends on N and approaches the normal distribution as N gets large.
standard deviation error limits  95% confidence limits
\[ (1-2 \times 0.025) \times 100 = 95\% \]

\[ \delta = tS_m = t \frac{\sigma}{\sqrt{N}} \]

\[ \Delta = t_{0.95} S_m = t_{0.95} \frac{\sigma}{\sqrt{N}} \]

(\( \Delta = 95\% \) confidence level in the mean.)

<table>
<thead>
<tr>
<th>( P' ) (%)</th>
<th>50</th>
<th>68.27</th>
<th>90</th>
<th>95</th>
<th>95.45</th>
<th>99</th>
<th>99.73</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu = 1  )</td>
<td>1.000</td>
<td>1.84</td>
<td>6.31</td>
<td>12.71</td>
<td>13.97</td>
<td>63.66</td>
<td>235.80</td>
</tr>
<tr>
<td>( \nu = 2  )</td>
<td>0.816</td>
<td>1.32</td>
<td>2.92</td>
<td>4.30</td>
<td>4.53</td>
<td>9.92</td>
<td>19.21</td>
</tr>
<tr>
<td>( \nu = 3  )</td>
<td>0.765</td>
<td>1.20</td>
<td>2.35</td>
<td>3.18</td>
<td>3.31</td>
<td>5.84</td>
<td>9.22</td>
</tr>
<tr>
<td>( \nu = 4  )</td>
<td>0.741</td>
<td>1.14</td>
<td>2.13</td>
<td>2.78</td>
<td>2.87</td>
<td>4.60</td>
<td>6.62</td>
</tr>
<tr>
<td>( \nu = 5  )</td>
<td>0.727</td>
<td>1.11</td>
<td>2.02</td>
<td>2.57</td>
<td>2.65</td>
<td>4.03</td>
<td>5.51</td>
</tr>
<tr>
<td>( \nu = 6  )</td>
<td>0.718</td>
<td>1.09</td>
<td>1.94</td>
<td>2.45</td>
<td>2.52</td>
<td>3.71</td>
<td>4.90</td>
</tr>
<tr>
<td>( \nu = 7  )</td>
<td>0.711</td>
<td>1.08</td>
<td>1.89</td>
<td>2.36</td>
<td>2.43</td>
<td>3.50</td>
<td>4.53</td>
</tr>
<tr>
<td>( \nu = 8  )</td>
<td>0.706</td>
<td>1.07</td>
<td>1.86</td>
<td>2.31</td>
<td>2.37</td>
<td>3.36</td>
<td>4.28</td>
</tr>
<tr>
<td>( \nu = 9  )</td>
<td>0.703</td>
<td>1.06</td>
<td>1.83</td>
<td>2.26</td>
<td>2.32</td>
<td>3.25</td>
<td>4.09</td>
</tr>
<tr>
<td>( \nu = 10 )</td>
<td>0.700</td>
<td>1.05</td>
<td>1.81</td>
<td>2.23</td>
<td>2.28</td>
<td>3.17</td>
<td>3.96</td>
</tr>
<tr>
<td>( \nu = 15 )</td>
<td>0.691</td>
<td>1.03</td>
<td>1.75</td>
<td>2.13</td>
<td>2.18</td>
<td>2.95</td>
<td>3.59</td>
</tr>
<tr>
<td>( \nu = 20 )</td>
<td>0.687</td>
<td>1.03</td>
<td>1.72</td>
<td>2.09</td>
<td>2.13</td>
<td>2.85</td>
<td>3.42</td>
</tr>
<tr>
<td>( \nu = 30 )</td>
<td>0.683</td>
<td>1.02</td>
<td>1.70</td>
<td>2.04</td>
<td>2.09</td>
<td>2.75</td>
<td>3.27</td>
</tr>
<tr>
<td>( \nu = 40 )</td>
<td>0.681</td>
<td>1.01</td>
<td>1.68</td>
<td>2.02</td>
<td>2.06</td>
<td>2.70</td>
<td>3.20</td>
</tr>
<tr>
<td>( \nu = 50 )</td>
<td>0.680</td>
<td>1.01</td>
<td>1.68</td>
<td>2.01</td>
<td>2.05</td>
<td>2.68</td>
<td>3.16</td>
</tr>
<tr>
<td>( \nu = 100 )</td>
<td>0.678</td>
<td>1.005</td>
<td>1.66</td>
<td>1.98</td>
<td>2.02</td>
<td>2.63</td>
<td>3.08</td>
</tr>
<tr>
<td>( \nu = \infty )</td>
<td>0.674</td>
<td>1.000</td>
<td>1.645</td>
<td>1.96</td>
<td>2.00</td>
<td>2.58</td>
<td>3.00</td>
</tr>
</tbody>
</table>
**Mean** = 12.26mL

\[ S_m = 1.01 / \sqrt{20} = 0.22 \]

\[ \Delta = t_{0.95}S_m \]

\[ \Delta = 2.09(0.22) = 0.460 = 0.46 \]

mean and \( S_m \) : 12.26 (0.22)mL

12.26 ± 0.46 mL (95%, N=20)
Mean = 12.42mL

\[
S_m = \frac{0.76}{\sqrt{255}} = 0.048 = 0.05
\]

\[
\Delta = t_{0.95}S_m
\]

\[
\Delta = 1.96(0.048) = 0.094 = 0.09
\]

\[
\text{mean and } S_m: \ 12.42 \ (0.05) \text{mL}
\]

\[
12.42 \pm 0.09 \text{mL} \quad (95\%, \ N=255)
\]

**Experimental Planning**

برنامه ریزی آزمایشات

ساده باشند
قابل تکرار باشند
عائي از خطاي سيستماتيک باشند
از دقت بالايى برخوردار باشند
گستره كاربردي وسيعي داشته باشند
هدفمند باشند
ارزان باشند
$$P = f(a, b)$$

At $b=40$

At $a=65$
روش‌های انجام آزمایشات

- روش سعی و خطا
  با تغییر تصادفی متغیرها و بررسی نشانه‌ها و بدنبال نتیجه شانسی بودن

- OVAT
  با تغییر یک متغیر با ثابت نگه داشتن نسبت به دوین نظر گرفتن اثرات یک متغیر

- روش فاکتوریل کامل Full Factorial
  انجام تست در تمام حالات ممکن برای متغیرها به روش یک متغیر

- روش فاکتوریل جزئی Fractional Factorial
  بررسی یکی از حالات ممکن همراه با بررسی اثرات یک متغیر که اطلاعات کمی دارد به فاکتوریل کامل حاصل می‌کند.
Joseph Hanak (1970)

The present approach to the search for new materials suffers from a chronic ailment, that of handling one sample at a time in the process of synthesis, chemical analysis and testing of properties. It is an expensive and time-consuming approach, which prevents highly-trained personnel from taking full advantage of its talents and keeps the tempo of discovery of new materials at a low level.

J.J. Hanak

(Journal of Materials Science 5, 964-971, 1970)
Combinatorial Methods in Drug Discovery

- Bruce Merrifield (1963) on solid-phase peptide synthesis (Nobel prize winner 1984)

- Early 1980s, Mario Gaysen; libraries of matrix 8x12

- 1985, Richard Houghten synthesised millions of peptides as mixtures and partially screened (first comprehensive combinatorial approach)
**Potentials in Materials Science**

Taking 50 metals (metal oxides)

19,600 ternary systems

537 million octernary

\[ N = \frac{(n+1)(n+2)}{2} \]

\[ n=10 \quad N=66 \]

---

**Artificial Intelligence**

Search refinement and steering (finding local optima)

**Relational Database**

- Artificial neural network
- Machine learning algorithm
- Steering software
Combinatorial Techniques
in Materials Science

- Thin Film Techniques
  - CVD
  - PVD with epitaxial growth

- Thick Film Method
  - Ink-jet Printing

Part of the phosphors library under 254 nm UV

Epitaxial Method
Ceramic mixtures
(sample library)
Design of Experiments

DOE

DOE = Structured Experimental Approach

برنامه ریزی ✓
انجام آزمایشات ✓
جمع آوری دیتا ✓
آналیز دیتا ✓
نتیجه گیری ✓

Full Factorial DOE

2-level experiment

\[ T = 2^k \]

\( k \) = number of factors

<table>
<thead>
<tr>
<th>TRIAL</th>
<th>FACTOR 1</th>
<th>FACTOR 2</th>
</tr>
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<tbody>
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بررسی و آنالیز نتایج آزمایشات

Practical
• آیا اختلاف داده ها معنی‌دار می‌باشد؟ و آیا روند خاصی در اختلاف‌ها دیده می‌شود؟

Graphical
• آیا با ترسیم نتایج الگویی در روند داده‌ها هدف می‌شود؟

Analytical
• کدام متغیر با متغیر دیگر تاثیر دارد؟
Strong Effect - Factor A

Effect of Factor A

Level of Factor A

Low

High

10

5

0

Effect of Factor B

Level of Factor B

Low

High

10

5

0

Strong Effect

No Effect

Strong Effect

Response

- Factor

Response

- Factor

- Factor


Effect of A-B Interaction

Level of Factor A

Low  High

B-Low  B-High

Strong Interaction

No Interaction
The 2³ Factorial Design

(a) Geometric view

(b) The design matrix

<table>
<thead>
<tr>
<th>Run</th>
<th>Factor</th>
<th>A</th>
<th>B</th>
<th>C</th>
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</table>

(a) Main effects
Table 6-3  Algebraic Signs for Calculating Effects in the 2^3 Design

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<tr>
<th>Treatment Combination</th>
<th>( I )</th>
<th>( A )</th>
<th>( B )</th>
<th>( AB )</th>
<th>( C )</th>
<th>( AC )</th>
<th>( BC )</th>
<th>( ABC )</th>
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<td>+</td>
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</tr>
<tr>
<td>( ab )</td>
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<td>-</td>
<td>-</td>
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<td>-</td>
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<tr>
<td>( c )</td>
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<td>+</td>
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</tr>
<tr>
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<td>+</td>
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</table>
Table 6-4  The Plasma Etch Experiment, Example 6-1

<table>
<thead>
<tr>
<th>Run</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Replicate 1</th>
<th>Replicate 2</th>
<th>Total</th>
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<td>−1</td>
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<td>604</td>
<td>(1) = 1154</td>
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<td>−1</td>
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<td>601</td>
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</table>

**Diagram:**

![3D diagrams](image)
Fractional Factorial

$2^{k-p}$ $3^{k-p}$

2-level standard experiments:

<table>
<thead>
<tr>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
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<tbody>
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</table>

Number of Factors
Table 8-2  The Two One-Half Fractions of the $2^3$ Design

<table>
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</table>

Full $2^2$ Factorial (Basic Design)

$2^3_{1-2}, I = ABC$

Table 8-9  Construction of the $2^5_1$ Design with the Generators $I = ABCE$ and $I = BCDF$

<table>
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<th>A</th>
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<th>C</th>
<th>D</th>
<th>E = ABC</th>
<th>F = BCD</th>
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