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INTEGRATING FAHP WITH COPRAS-G METHOD FOR SUPPLIER SELECTION (CASE STUDY: AN IRANIAN MANUFACTURING COMPANY)

Mohammadsadegh Mobin*
Afshan Roshani
Western New England University, Springfield, MA, USA

Mahdi Saeedpoor
Allameh Tabataba’i University, Tehran, Iran

Mohammad Mahdi Mozaffari
Imam Khomeini International University, Qazvin, Iran

*mm337076@wne.edu

Abstract
Supplier selection has become a core competency for many companies, however, the nature of these decisions can be complex and unstructured. This paper presents a fuzzy-Grey decision-making approach designed for supplier selection which is generally considered a Multi-Criteria Decision Making (MCDM) problem. As is the nature of real world MCDM application, both quantitative and qualitative factors must be considered along with the feedback of decision makers. The introduction of expert opinion requires a robust methodology since their judgments and preferences are often vague and cannot be easily estimated with a numerical value.

In this paper, an integration of a Fuzzy Analytic Hierarchy Process (FAHP) and the Complex Proportional Assessment of alternatives to Grey relations (CORPAS-G) is proposed to prioritize suppliers in an Iranian manufacturing industry. Commonly cited criteria for supplier selection are identified via literature review and selected by experts. Linguistic values which are expressed in triangular fuzzy numbers are then used to assess the ratings for criteria which are weighted based on the applied AHP model and fuzzy logic. After obtaining each criterion’s weight, suppliers are ranked based on the COPRAS-G method. Lastly, advantages of using fuzzy and grey values to tackle uncertainty in decision makers’ judgments are discussed.

Keywords
Supplier selection, Fuzzy Analytic Hierarchy Process (FAHP), COPRAS-G, Multi-Criteria Decision Making (MCDM).

Introduction
With the globalization of the economic market and the development of information technology, many companies consider that a well-designed and implemented supply chain management (SCM) system is an important tool for increasing competitive advantage. Consequently, the supplier selection problem becomes one of the most important components in SCM. In the past, several methods have been proposed to solve the supplier selection problem. Main methods have been the Analytic Hierarchy Process (AHP) (Chan, 2003), the Analytic Network Process (Gencer & Gürpinar, 2007), Data Envelopment Analysis (DEA) (Noorizadeh, Mahdiloo, & Saen, 2012), and Mathematical Programming (MP) techniques (Chaudhry, Forst, & Zydiak, 1993). Although linear weighting is a very simple method, it depends heavily on human judgments and also weights the criteria equally, which rarely happens in practice. Furthermore, MP techniques cause a significant problem in considering qualitative factors. These disadvantages of linear weighting and MP have led to an increase in MCDM application in recent years.

Supplier selection can be modeled as a Multiple-Criteria Decision Making (MCDM) problem involving various tangible and intangible factors. Decision makers (DMs) express their expert opinion regarding those factors...
to formulate selection criteria. Next, the criteria are applied to a pre-determined set of supplier alternatives with the intention of ranking them. Following ranking, the most desirable alternative is chosen. In conventional MCDM methods, the ratings and weights of the criteria are known precisely. However, DM judgment is often uncertain and cannot be estimated by an exact numerical value. Thus, the problem of selecting suppliers has many uncertainties and becomes more difficult.

To combat uncertainty, Fuzzy theory (Zadeh, 1965) can be applied to the supplier selection problem. Such methods integrating fuzzy theory with MCDM for supplier selection include Fuzzy AHP (FAHP) (Kilincci & Onal, 2011), Fuzzy ANP (Vinodh, Anesh Rамиya, & Gautham, 2011), Fuzzy TOPSIS (Bottani & Rizzi, 2006), Fuzzy AHP and VIKOR (Valehzaghard, Mozaffari & Valehzagharad, 2011), etc. In addition to fuzzy theory, Grey theory (Deng, 1989) can also be applied to problems involving DM uncertainty as it displays superiority in the mathematical analysis of systems with uncertain information. Research in this area include a grey-based MCDM approach done by Li et al. (Li, Yamaguchi, & Nagai, 2007). Others include supplier selection model based on the grey system theory (Huixia & Tao, 2008) and grey TOPSIS (Jadidi, Yusuff, Firouzi, & Hong, 2008).

By performing a thorough literature review, it was discovered that no integrated MCDM method which includes the fuzzy and grey theories simultaneously has been used in the context of supplier selection. Therefore, this paper aims to fill that gap by using fuzzy numbers in AHP to weight criteria and then grey numbers in COPRAS to rank the alternatives in a supplier selection problem with uncertainty due to human element associated with expert judgment.

For supplier selection, both quantitative and qualitative criteria are required. Lima Junior et al. (Lima Junior, Osiro, & Carpinetti, 2014) summarized the most used criteria for supplier selection from 14 prior studies. Some of the most popular criteria used include the following: technical capability, quality, flexibility (response to change), cost/price, financial situation, reputation, easy of communication, on time delivery, relationship, product performance, after sale warranty, and geographic location. Based on expert judgment, the following six criteria are used in this study; C1: Quality (commitment to quality and quality of conformance), C2: Price, C3: On time delivery, C4: Technical capability, C5: Flexibility (response to change) and C6: Reputation.

To further examine the use of Fuzzy AHP and Grey Theory in MCDM, the remainder of this paper is organized as follows. Next section presents the proposed integrated method for supplier selection. This method includes fuzzy AHP for weighting the criteria and COPRAS-G method to prioritize the suppliers. Afterwards, a case study of supplier selection process in an Iranian manufacturing company is presented. The results of applying the proposed integrated MCDM method are presented in last section.

**Proposed model for supplier selection problem**

**Fuzzy Analytic Hierarchy Process**

An effective DM model needs to consider vagueness and ambiguity since both are common in real-world decision making. Problems arise in traditional methods during the transformation of qualitative preferences to point estimates since DMs often provide uncertain answers rather than precise values (Wang, Luo, & Hua, 2008). Conventional AHP that requires the selection of subjective values in pairwise comparison may not be sufficient and uncertainty should be considered in some or all pairwise comparison values (Kabir, 2012). Kabir and Hasin (Kabir & Hasin, 2011) summarize the shortcomings of AHP are as follows: (1) the AHP method is mainly used in nearly crisp decision applications, (2) the AHP method does not take into account the uncertainty associated with the mapping of one's judgment to a number, (3) the subjective judgment, selection, and preference of DMs have great influence on the AHP results, (4) ranking of the AHP method is rather imprecise, (5) the AHP method creates and deals with a very unbalanced scale of judgment. Therefore, conventional AHP seems inadequate to capture DMs requirements explicitly.

In practice, expert judgment is typically more consistent when DMs are able to give interval responses rather than fixed value responses. Since the fuzzy linguistic approach can take the pessimism/optimism rating attitude of DMs into account, linguistic values whose membership functions are usually characterized by triangular or trapezoidal fuzzy numbers, are recommended to assess preference ratings instead of conventional numerical equivalence method (Kabir, 2012). As a result, in an uncertain pairwise comparison environment, fuzzy AHP should be more appropriate and effective than conventional AHP (Wang et al., 2008).

Researchers propose several methods such as geometric mean, fuzzy modification of the logarithmic least squares, fuzzy least square, and direct fuzzification methods to attain the priorities in FAHP (Kabir & Akhtar Hasin, 2012). Chang (Chang, 1996) developed a new approach for handling FAHP using Triangular Fuzzy Numbers (TFN) for the pairwise comparison scale of FAHP and the extent analysis method for the synthetic extent values of the
pairwise comparisons. A TFN denoted as \( \tilde{M} = (l, m, u) \) where \( l < m < u \), has the following triangular membership function (Eq.1):

\[
\mu_{\tilde{M}}(x) = \begin{cases} 
0, & x < l \\
\frac{x - 1}{m - l}, & l \leq x \leq m \\
\frac{u - x}{u - m}, & m \leq x \leq u \\
0, & x > u 
\end{cases}
\] (1)

In this methodology, DMs are asked to indicate the relative importance of two evaluation criteria in the same level. Linguistic pairwise comparisons are transformed to corresponding TFNs illustrated in Exhibit 1. In this research, the table proposed by Kaya and Kahraman (Kaya & Kahraman, 2011) was used to convert linguistic terms to fuzzy scales. DMs used the linguistic variables in Exhibit 1 to evaluate the importance of the criteria.

**Exhibit 1. Linguistic Variables and Fuzzy Scales (Kaya & Kahraman, 2011).**

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>Fuzzy score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolutely strong (AS)</td>
<td>(2, 5/2, 3)</td>
</tr>
<tr>
<td>Very strong (VS)</td>
<td>(3/2, 2, 5/2)</td>
</tr>
<tr>
<td>Fairly strong (FS)</td>
<td>(1, 3/2, 2)</td>
</tr>
<tr>
<td>Slightly strong (SS)</td>
<td>(1, 1, 3/2)</td>
</tr>
<tr>
<td>Equal (E)</td>
<td>(1, 1, 1)</td>
</tr>
<tr>
<td>Slightly weak (SW)</td>
<td>(2/3, 1, 1)</td>
</tr>
<tr>
<td>Fairly weak (FW)</td>
<td>(1/2, 2/3, 1)</td>
</tr>
<tr>
<td>Very Weak (VW)</td>
<td>(2/5, 1/2, 2/3)</td>
</tr>
<tr>
<td>Absolutely weak (AW)</td>
<td>(1/3, 2/5, 1/2)</td>
</tr>
</tbody>
</table>

Geometric mean operations (Eq. 2) are used to combine pairwise comparisons of DMs or aggregating group decisions (Davies, 1994):

\[
l_{ij} = \left( \prod_{k=1}^{K} l_{ijk} \right)^{1/k}, \quad m_{ij} = \left( \prod_{k=1}^{K} m_{ijk} \right)^{1/k}, \quad u_{ij} = \left( \prod_{k=1}^{K} u_{ijk} \right)^{1/k}
\] (2)

After aggregating group decisions, it is necessary to check the Consistency Ratio (CR) of the comparison. In this method, graded mean integration approach is utilized for defuzzifying the matrix. According to the graded mean integration approach, a fuzzy number \( \tilde{M} \) can be transformed into a crisp number by employing Eq.3 (Kabir, 2012):

\[
P(\tilde{M}) = M = \frac{l + 4m + u}{6}
\] (3)

After the defuzzification of each value in the matrix, CR of the matrix is calculated using randomly generated Consistency Index (CI) given in Exhibit 2. Next, CR is checked to see whether it is smaller than 0.10 or not.

**Exhibit 2. Randomly Generated Consistency Index (CI) for Different Sizes of Matrix (Kabir & Hasin, 2011).**

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.I.</td>
<td>0</td>
<td>0</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
<td>1.49</td>
</tr>
</tbody>
</table>

In this study, modified Chang extent analysis method proposed by Kabir and Hasin (Kabir & Hasin, 2011) was used to derive the importance weights of criteria from pairwise comparisons. Let \( W \) be the normalized weight vector of triangular fuzzy comparison matrix, which includes the importance weights of criteria in the crisp form. The steps for calculating this vector are as follows (Kabir, 2012; Kabir & Hasin, 2012): First, calculate the fuzzy synthetic extent value of each pairwise comparison. Denote \( \tilde{C}_{ij} \) as the TFN related to the pairwise comparison of
criterion $i$ over criterion $j$, which is represented as $(l_{ij}, m_{ij}, u_{ij})$. According to Chang (Chang, 1996), the value of fuzzy synthetic extent with respect to the criterion $i$, denoted as $S_i = (l_i, m_i, u_i)$, can be obtained via Eq.4, where $n$ is the size of the fuzzy judgment matrix.

$$S_i = \sum_{j=1}^{n} C_{ij} \otimes [\sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij}]^{-1}, i = 1,2,\ldots,n \tag{4}$$

To obtain $\sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij}$, fuzzy addition operation of $C_{ij}$ values can be performed as Eq.5:

$$\sum_{j=1}^{n} C_{ij} = \left(\sum_{j=1}^{n} l_{ij}, \sum_{j=1}^{n} m_{ij}, \sum_{j=1}^{n} u_{ij}\right), \quad i = 1,2,\ldots,n$$

The inverse vector of the Eq.5 can be determined by using Eq.6:

$$[\sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij}]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} \sum_{j=1}^{n} l_{ij}}, \frac{1}{\sum_{i=1}^{n} \sum_{j=1}^{n} m_{ij}}, \frac{1}{\sum_{i=1}^{n} \sum_{j=1}^{n} u_{ij}}\right) \tag{6}$$

Wang et al. (Wang et al., 2008) corrected the normalization formula (Eq.6) as Eq.7.

$$[\sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{C}_{ij}]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} l_{ij} + \sum_{k=1,k\neq i}^{n} u_{kj}}, \frac{1}{\sum_{i=1}^{n} m_{ij} + \sum_{k=1,k\neq i}^{n} u_{kj}}\right) \tag{7}$$

The second step is to derive fuzzy ranking value of $\tilde{S}_i$. In this step, $\tilde{S}_i$ is compared to other synthetic extent values of $A$, $\tilde{S}_j = (l_j, m_j, u_j)$. According to Chang (Chang, 1996), the degree of possibility of $\tilde{S}_i \geq \tilde{S}_j$ is obtained by Eq.8:

$$V(S_j \geq S_i) = \text{height}(S_i \cap S_j) = \mu_{\tilde{S}_i}(d) = \begin{cases} 1, & \text{if } m_j \geq m_i \\ 0, & \text{if } l_i \geq u_j \\ \frac{l_i - u_j}{(m_j - u_j) - (m_i - l_i)}, & \text{otherwise} \end{cases} \tag{8}$$

where $d$ is the ordinate of the highest intersection point $D$ between $\mu_{\tilde{S}_i}$ and $\mu_{\tilde{S}_j}$ as shown in Exhibit 3. To compare $\tilde{S}_i$ and $\tilde{S}_j$, both the values of $V(S_j \geq S_i)$ and $V(S_i \geq S_j)$ are needed.

**Exhibit 3. The Intersection Between $\tilde{S}_i$ and $\tilde{S}_j$ (Kabir & Hasin, 2011).**

The degree of possibility defined by the extent analysis method is an index for comparing two TFNs rather than an index for calculating their relative importance (Chang, 1996). As a result, normalized degrees of possibility can only show to what degree a TFN is greater than all the others. It cannot be used to represent their relative importance (Kabir, 2012; Chang, 1996). To resolve this problem, Liou and Wang (Liou & Wang, 1992) proposed total integral value with index of optimism, which derives the priorities of the synthetic extent values by Eq.9 (Kabir, 2012):
where $\alpha$ is the index of optimism which represents degree of optimism for decision makers. If $\alpha$ approaches 1 in [0, 1], the decision-makers are more optimistic. Otherwise they are considered more pessimistic. Finally, the normalized importance weight vector $W = (w_1, w_2, ..., w_n)^T$ of the fuzzy judgment matrix is determined by using Eq.10, where $w_i$ is a non-fuzzy number.

$$w_i = \frac{I_F^i(\tilde{s}_i)}{\sum_{i=1}^{n} I_F^i(\tilde{s}_i)}, \text{ } i = 1, 2, ..., n$$ (10)

**COPRAS-G method**

The COPRAS-G (Complex Proportional Assessment of alternatives with Grey relations) method proposed by Zavadskas et al. (E K Zavadskas, 2008; Zavadskas, Kaklauskas, Turskis, & Tamošaitiene, 2008) is widely used to calculate the utility degree and priority order of the alternatives. This method uses a stepwise ranking and evaluation procedure for the alternatives in terms of their significance and utility degree. As described earlier, the criteria values used in this ranking of alternatives are expressed in terms of fuzzy numbers (e.g. C.-T. Chen, Lin, & Huang, 2006) or in interval values (e.g. Li et al., 2007; Huixia & Tao, 2008).

The theory of a grey system, introduced by Deng (Deng, 1989), is applied to convert crisp values (white numbers) to grey numbers. The word grey is used to classify the degree to which information in the system is known. For instance, if the system information is fully known, the system is called a white system. If the information is unknown, it is called a black system. Similarly, a system with information partially known is called a grey system (Li et al., 2007). A grey number (denoted by $\otimes$) is the basic element in grey systems used to describe the uncertain information. Let $\otimes X = [x, \bar{x}] = \{x | x \leq \bar{x} \leq \bar{x} \}$ and $x \in R$, then $\otimes X$ which has two real numbers $x$ (the lowest limit of $\otimes X$) and $\bar{x}$ (the upper limit of $\otimes X$) is defined as follows (Li et al., 2007):

- If $x \rightarrow -\infty$ and $\bar{x} \rightarrow -\infty$, then $\otimes X$ is called the black number having no meaningful information.
- Else if $x = \bar{x}$, then $\otimes X$ is called the white number with the complete information.
- Otherwise, $\otimes X = [x, \bar{x}]$ is called the grey number which has insufficient and uncertain information.

Therefore, the decision makers’ judgments about suppliers, which accommodate an uncertain level of information, can be described by the grey system through the classification of white, black, and grey numbers (Deng, 1989). The main advantage of grey theory over fuzzy theory is that grey theory considers the condition of the fuzziness. That is, grey theory can deal flexibly with the fuzziness of the situation (Li et al., 2007). Other advantages of the Grey theory over fuzzy theory are mentioned by Zavadskas (E K Zavadskas, 2008).

The COPRAS-G method suggested by Zavadskas et al. (2008) (E K Zavadskas, 2008; Zavadskas et al., 2008) expresses attributes in interval form which is suitable for grey theory application. This approach is a newly developed Multiple Attribute Decision Making (MADM) process for evaluating the alternatives. It is completely logical and useful mathematically for processing incomplete information about the system (E K Zavadskas, 2008) and is intended to increase the efficiency and improve the accuracy level of the resolution process in the decision-making process. In COPRAS-G, the degree of utility of an alternative is shown as a percentage which illustrates the degree to which one alternative is considered better or worse than the other existing alternatives. It estimates the market value of alternatives and gathers diverse recommendations. Other MADM approaches do not have such features, making COPRAS-G valuable in the decision-making process (Skeete, Mobin & Salmon, 2015). COPRAS-G supports the decision makers to make more accurate decisions and effectively handles problems associated with uncertainty, subjectivity and imprecise data. The procedure of COPRAS-G is presented in a few articles such as (E K Zavadskas, 2008; Bindu Madhuri, Ch, 2010; Zavadskas, Turskis, & Tamošaitiene, 2010). The COPRAS-G method is summarized below.

Step 1: Identify the relevant criteria to the decision making problem.

Step 2: Construct the decision matrix $\otimes X$ as follow:

$$\otimes X = \begin{bmatrix}
\otimes x_{11} & \otimes x_{12} & \ldots & \otimes x_{1m} \\
\otimes x_{21} & \otimes x_{22} & \ldots & \otimes x_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\otimes x_{n1} & \otimes x_{n2} & \ldots & \otimes x_{nm}
\end{bmatrix} = \begin{bmatrix}
\underline{x}_{11};\bar{x}_{11} & \ldots & \underline{x}_{11};\bar{x}_{11} \\
\underline{x}_{21};\bar{x}_{21} & \ldots & \underline{x}_{2m};\bar{x}_{2m} \\
\vdots & \ddots & \vdots \\
\underline{x}_{n1};\bar{x}_{n1} & \ldots & \underline{x}_{nm};\bar{x}_{nm}
\end{bmatrix}$$ (11)
where ⊗ \( x_{ij} \) is determined by \( \bar{x}_{ji} \) (the smallest value, i.e., the lower limit) and \( \bar{x}_{ji} \) (the biggest value, i.e., the upper limit). This research used Exhibit 4 proposed by Nguyen et al., (2014) (Nguyen, Dawal, Nukman, & Aoyama, 2014) to convert linguistic variables to grey numbers for evaluating the alternatives.

**Exhibit 4.** Linguistic Variables and Equivalent Grey Numbers for Evaluating the Alternatives.

<table>
<thead>
<tr>
<th>Linguistic variables</th>
<th>Grey numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Weak (VW)</td>
<td>[1, 2]</td>
</tr>
<tr>
<td>Weak (W)</td>
<td>[2, 4]</td>
</tr>
<tr>
<td>Medium (M)</td>
<td>[4, 6]</td>
</tr>
<tr>
<td>Good (G)</td>
<td>[6, 8]</td>
</tr>
<tr>
<td>Very Good (VG)</td>
<td>[8, 9]</td>
</tr>
</tbody>
</table>

Step 3: Determine the importance weight of each criterion. In this case, Fuzzy AHP method was used to obtain the weights of criteria.

Step 4: Calculate the weighted normalized decision matrix. In this step, the decision-making matrix \( \otimes X \) was normalized in order to determine the importance weight of the selection criteria:

\[
\bar{x}_{ji} = \frac{1}{2} \left( \sum_{j=1}^{n} x_{ji} + \sum_{j=1}^{n} \bar{x}_{ji} \right) \quad \text{and} \quad \bar{x}_{ji} = \frac{1}{2} \left( \sum_{j=1}^{n} \bar{x}_{ji} + \sum_{j=1}^{n} \bar{x}_{ji} \right)
\]

(12)

where \( j = 1, n; i, m \). \( x_{ji} \) is the lowest value of criterion \( i \) for alternative \( j \), \( \bar{x}_{ji} \) is the highest value of criterion \( i \) for alternative \( j \), \( m \) is the number of criteria, and \( n \) is the number of alternatives under consideration. The normalization process results in the following normalized decision matrix (Eq. 13):

\[
\otimes \bar{X} = \begin{bmatrix}
\bar{x}_{11} & \bar{x}_{12} & \ldots & \bar{x}_{1m} \\
\bar{x}_{21} & \bar{x}_{22} & \ldots & \bar{x}_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\bar{x}_{n1} & \bar{x}_{n2} & \ldots & \bar{x}_{nm}
\end{bmatrix}
\]

(13)

In order to construct the weighted normalized decision matrix, first calculate the weighted normalized values \( \otimes \bar{x}_{ji} \) as Eq. 14, where \( q_i \) is the relative importance of the \( i \)th criterion determine by using the FAHP.

\[
\otimes \bar{x}_{ji} = \otimes \bar{x}_{ji} \cdot q_i \text{ or } \bar{x}_{ji} = \bar{x}_{ji} \cdot q_i \text{ and } \bar{x}_{ji} = \bar{x}_{ji} \cdot q_i
\]

(14)

Then, the weighted normalized decision matrix \( \otimes \bar{X} \) is constructed as Eq. 15:

\[
\otimes \bar{X} = \begin{bmatrix}
\otimes \bar{x}_{11} & \otimes \bar{x}_{12} & \ldots & \otimes \bar{x}_{1m} \\
\otimes \bar{x}_{21} & \otimes \bar{x}_{22} & \ldots & \otimes \bar{x}_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\otimes \bar{x}_{n1} & \otimes \bar{x}_{n2} & \ldots & \otimes \bar{x}_{nm}
\end{bmatrix} = \begin{bmatrix}
\bar{x}_{11} & \bar{x}_{12} & \ldots & \bar{x}_{1m} \\
\bar{x}_{21} & \bar{x}_{22} & \ldots & \bar{x}_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\bar{x}_{n1} & \bar{x}_{n2} & \ldots & \bar{x}_{nm}
\end{bmatrix} \cdot \begin{bmatrix}
q_1 & \bar{x}_{11} & \bar{x}_{12} & \ldots & \bar{x}_{1m} \\
\bar{x}_{21} & q_2 & \bar{x}_{22} & \ldots & \bar{x}_{2m} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\bar{x}_{n1} & \bar{x}_{n2} & \ldots & q_n & \bar{x}_{nm}
\end{bmatrix}; j = 1, n; i, m
\]

(15)

Step 5: Determine the relative significance of each alternative: First, calculate the sums \( P_j \) of the criterion values (whose larger values are more preferable) as Eq.16:

\[
P_j = \frac{1}{2} \sum_{i=1}^{k} (\bar{x}_{ji} + \bar{x}_{ji})
\]

(16)
Then, calculate the sums $R_j$ of the criterion values (whose smaller values are more preferable) as Eq.17, where $(m - k)$ is the number of criteria which must be minimized. Next, determine the minimum value of $R_j$ (Eq.18). The relative significance of each alternative is then calculated as Eq.19.

$$R_j = \frac{1}{2} \sum_{i=k+1}^{m} (\tilde{x}_{ji} + \tilde{x}_{ji}) ; i = k,m$$

$$R_{\min} = \min_j R_j ; j = 1,n$$

$$Q_j = P_j + \frac{\sum_{j=1}^{n} R_j}{R_j \sum_{j=1}^{n} 1}$$

Step 6: Calculate the utility degree of each alternative: In order to calculate the utility degree of each alternative, first determine the optimally criterion $K$ as Eq.20:

$$K = \max_j Q_j ; j = 1,n$$

The degree of project utility is determined by comparing the alternatives under consideration with the best alternative. The values of the utility degree range from 0% (for the worst alternative) to 100% (for the best alternative). The utility degree of each alternative $j$ is calculated as Eq.21:

$$N_j = \frac{Q_j}{Q_{\max}} \times 100\%$$

**Case study**

The proposed fuzzy-grey integrated MCDM method was applied to select suppliers of an Iranian company which produces mineral water. As mentioned earlier, the final criteria were selected based on experts’ judgment from the most commonly used criteria found in literature review. The final criteria included: quality (commitment to quality and quality of conformance), price, on time delivery, technical capability, flexibility (response to change) and reputation. Three suppliers were considered in this decision making process. The following sections, present the process of applying the proposed integrated MCDM method.

**Applying FAHP to determine the weights of criteria**

Pairwise comparisons of the criteria were made by 5 experts from top level management of the organization. The pairwise comparisons of criteria were in linguistic terms. As it is not possible to make arithmetical operations with linguistic terms, each term was associated with a triangular fuzzy number using Exhibit 1. Next, using Exhibit 1 and the linguistic values of pairwise comparisons, the fuzzy evaluation matrix for the criteria weights was obtained (Exhibit 5). In order to obtain this matrix, the arithmetic means of the fuzzy scores were calculated (Eq.2).

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>(1.00,1.00,1.00)</td>
<td>(0.84,1.00,1.08)</td>
<td>(1.08,1.35,1.71)</td>
<td>(1.08,1.24,1.75)</td>
<td>(1.00,1.08,1.46)</td>
<td>(1.08,1.35,1.86)</td>
</tr>
<tr>
<td>C2</td>
<td>(0.92,1.00,1.17)</td>
<td>(1.00,1.00,1.00)</td>
<td>(1.31,1.84,2.35)</td>
<td>(1.31,1.69,2.22)</td>
<td>(1.00,1.27,1.78)</td>
<td>(1.31,1.69,2.22)</td>
</tr>
<tr>
<td>C3</td>
<td>(0.58,0.74,0.92)</td>
<td>(0.42,0.54,0.75)</td>
<td>(1.00,1.00,1.00)</td>
<td>(1.00,1.17,1.68)</td>
<td>(0.69,0.84,1.08)</td>
<td>(0.73,0.92,1.08)</td>
</tr>
<tr>
<td>C4</td>
<td>(0.56,0.80,0.92)</td>
<td>(0.45,0.58,0.75)</td>
<td>(0.59,0.85,1.00)</td>
<td>(1.00,1.00,1.00)</td>
<td>(0.84,1.00,1.17)</td>
<td>(1.00,1.17,1.55)</td>
</tr>
<tr>
<td>C5</td>
<td>(0.68,0.92,1.00)</td>
<td>(0.56,0.78,1.00)</td>
<td>(0.92,1.17,1.43)</td>
<td>(0.85,1.00,1.17)</td>
<td>(1.00,1.00,1.00)</td>
<td>(1.24,1.62,2.14)</td>
</tr>
<tr>
<td>C6</td>
<td>(0.53,0.74,0.92)</td>
<td>(0.45,0.58,0.75)</td>
<td>(0.92,1.08,1.35)</td>
<td>(0.64,0.85,1.00)</td>
<td>(0.46,0.61,0.80)</td>
<td>(1.00,1.00,1.00)</td>
</tr>
</tbody>
</table>

In order to check the Consistency Ratio (CR) of the evaluation matrix, the graded mean integration approach (Eq.3) was utilized for defuzzification. The CR value for the defuzzified version of the evaluation matrix was calculated as 0.89. Since this is less than 1.24, the comparison results can be considered consistent and suitable for an AHP procedure. Next, using Eq.4, Eq.5 and Eq.7 fuzzy synthetic extent value ($\tilde{S}_i$) for the criteria was
produced. After obtaining the synthetic extent values, Eq.9 was used for calculating the priorities of the synthetic extent values when the index of optimism ($\alpha$) was considered as 0.5. Finally, via normalization (Eq.10), the normalized importance weight vector of the fuzzy judgment matrix was obtained as in Exhibit 6.

**Exhibit 6.** Results of the AHP Fuzzy Procedure for the Determination of the Weights.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$S_i = (l_i, m_i, u_i)$</th>
<th>$I^\alpha(\tilde{S}_i)$</th>
<th>$w_i$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>(0.2618, 0.1871, 0.1407)</td>
<td>0.1942</td>
<td>0.1903</td>
<td>2</td>
</tr>
<tr>
<td>C2</td>
<td>(0.3068, 0.2264, 0.1627)</td>
<td>0.2306</td>
<td>0.2260</td>
<td>1</td>
</tr>
<tr>
<td>C3</td>
<td>(0.1964, 0.1392, 0.1008)</td>
<td>0.1439</td>
<td>0.1410</td>
<td>5</td>
</tr>
<tr>
<td>C4</td>
<td>(0.1935, 0.1442, 0.1009)</td>
<td>0.1457</td>
<td>0.1428</td>
<td>4</td>
</tr>
<tr>
<td>C5</td>
<td>(0.2302, 0.1731, 0.1205)</td>
<td>0.1742</td>
<td>0.1707</td>
<td>3</td>
</tr>
<tr>
<td>C6</td>
<td>(0.1768, 0.1298, 0.0906)</td>
<td>0.1318</td>
<td>0.1292</td>
<td>6</td>
</tr>
</tbody>
</table>

Applying CORPRAS-G method to prioritize suppliers

After obtaining the weights of criteria, the next step was the determination of the best supplier. To do this, five experts evaluated the suppliers with respect to each criterion. Evaluation results are given in Exhibit 7. The linguistic values in Exhibit 7 were converted to grey numbers, which are shown in Exhibit 2. The average of experts’ judgment for each alternative was calculated using the grey theory operator which is the same as arithmetic mean. The decision support matrix was obtained and is shown in Exhibit 8.

**Exhibit 7.** The Experts’ Judgment for Evaluating the Alternatives.

<table>
<thead>
<tr>
<th>Experts</th>
<th>Suppliers</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. 1</td>
<td>Alt. 1</td>
<td>G</td>
<td>M</td>
<td>G</td>
<td>M</td>
<td>VG</td>
<td>L</td>
</tr>
<tr>
<td>Exp. 2</td>
<td>Alt. 1</td>
<td>G</td>
<td>M</td>
<td>G</td>
<td>M</td>
<td>VG</td>
<td>M</td>
</tr>
<tr>
<td>Exp. 3</td>
<td>Alt. 1</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>VG</td>
<td>M</td>
</tr>
<tr>
<td>Exp. 4</td>
<td>Alt. 1</td>
<td>M</td>
<td>M</td>
<td>G</td>
<td>L</td>
<td>G</td>
<td>M</td>
</tr>
<tr>
<td>Exp. 5</td>
<td>Alt. 1</td>
<td>VG</td>
<td>M</td>
<td>G</td>
<td>M</td>
<td>VG</td>
<td>M</td>
</tr>
</tbody>
</table>

**Exhibit 8.** The Decision Support Matrix for Alternatives with Grey Numbers.

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alt. 1</td>
<td>[5.6, 7.4]</td>
<td>[4.0, 6.0]</td>
<td>[5.6, 7.6]</td>
<td>[3.6, 5.6]</td>
<td>[7.6, 8.8]</td>
<td>[3.6, 5.6]</td>
</tr>
<tr>
<td>Alt. 2</td>
<td>[8.0, 9.0]</td>
<td>[4.0, 6.0]</td>
<td>[5.6, 7.6]</td>
<td>[7.6, 8.8]</td>
<td>[6.8, 8.4]</td>
<td>[7.2, 8.6]</td>
</tr>
<tr>
<td>Alt. 3</td>
<td>[5.6, 7.6]</td>
<td>[8.0, 9.0]</td>
<td>[4.0, 6.0]</td>
<td>[5.2, 7.2]</td>
<td>[7.2, 8.6]</td>
<td>[4.8, 6.6]</td>
</tr>
</tbody>
</table>

The normalization of the data in the decision matrix was determined by Eq.12. The weighted normalized decision support matrix was calculated through Eq.24. These results are depicted in Exhibit 9.

**Exhibit 9.** The Weighted Normalized Decision Support Matrix for the Alternatives.

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alt. 1</td>
<td>[0.049, 0.065]</td>
<td>[0.049, 0.073]</td>
<td>[0.044, 0.060]</td>
<td>[0.027, 0.042]</td>
<td>[0.055, 0.063]</td>
<td>[0.026, 0.040]</td>
</tr>
<tr>
<td>Alt. 2</td>
<td>[0.070, 0.079]</td>
<td>[0.049, 0.073]</td>
<td>[0.044, 0.060]</td>
<td>[0.057, 0.066]</td>
<td>[0.049, 0.060]</td>
<td>[0.051, 0.061]</td>
</tr>
<tr>
<td>Alt. 3</td>
<td>[0.049, 0.067]</td>
<td>[0.097, 0.109]</td>
<td>[0.031, 0.047]</td>
<td>[0.039, 0.054]</td>
<td>[0.052, 0.062]</td>
<td>[0.034, 0.047]</td>
</tr>
</tbody>
</table>
The sums $P_j$ (Eq. 16) and $R_j$ (Eq. 17) of the criterion values were calculated to obtain the relative significance of each alternative ($Q_j$). Finally, the utility degree of each alternative ($N_j$) was determined by Eq.21. Results are presented in Exhibit 10 which shows the priority order of suppliers as Supplier 2 > Supplier 1 > Supplier 3.

**Exhibit 10. The Results of COPRAS-G Method.**

<table>
<thead>
<tr>
<th>Rank</th>
<th>Alt. 1</th>
<th>Alt. 2</th>
<th>Alt. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_j$</td>
<td>$R_j$</td>
<td>$Q_j$</td>
</tr>
<tr>
<td>2</td>
<td>0.0608</td>
<td>0.2350</td>
<td>0.3220</td>
</tr>
<tr>
<td>1</td>
<td>0.0608</td>
<td>0.2988</td>
<td>0.3857</td>
</tr>
<tr>
<td>3</td>
<td>0.1033</td>
<td>0.2410</td>
<td>0.2921</td>
</tr>
</tbody>
</table>

**Conclusions**

Supplier selection involving both qualitative and quantitative factors can be modeled as a Multi-Criteria Decision Making problem. In this research, the hybrid approach of fuzzy AHP and COPRAS-G method has been proven to provide an effective decision when evaluating suitable suppliers. A case study was performed at an Iranian company utilizing this research method. In it, a team of 5 experts worked together to determine the decision criteria and suggested a set of potential suppliers to consider as alternatives. Fuzzy AHP was then used to obtain the weight of each criterion, and finally COPRAS-G method was applied to prioritize the suppliers. The results of applying this integrated MCDM method in case study show that based on experts’ judgments, four highest priority, most important criteria for supplier selection are price (C2), quality (C1), flexibility (C5), and technical capability (C4). The result of applying COPRAS-G method shows the priority of alternatives as follows: Supplier 2 > Supplier 1 > Supplier 3.

This integrated methodology is beneficial for decision makers since it uses fuzzy logic capable of addressing the imprecise, vague, and uncertain information that stems for decision makers’ judgments. Furthermore, the COPRAS-G method allows experts to express criteria information in interval values rather than scalar estimation. It is also used to obtain the final ranking of the alternatives.

This research may be extended in a number of ways. First, Analytic Network Process (ANP) can be employed for cases where interaction exists between criteria. Similar to AHP, ANP weights criteria and relies on expert opinion. This requires fuzzy logic to be applied similarly to the methodology proposed in this paper. Second, the selection of experts themselves can be analyzed. It is imperative that participants have an appropriate level of experience and knowledge.

**References**


**About the Authors**

Mohammad Sadegh Mobin is a doctoral student in Industrial Engineering and Engineering Management at the Western New England University, MA, USA. He holds a Master degree in Operations Research (OR) (2011) and a bachelor degree in Industrial Engineering (IE) (2009). He served as a quality engineer (2006-2012) in different industries. His research interests lie in the areas of different applications of operations research tools.

Afshan Roshani is a 1st year graduate student in Industrial Engineering and Engineering Management at the Western New England University. She holds an MBA (2012) and a bachelor degree in IE (2009). She works in different industries as a quality engineer (2007-2013). Her current research activates are centered in her interest in quality engineering and supply chain.

Mahdi Saeedpoor is currently HR expert in Goldiran Co., Iran. He received master’s degree in industrial management (2011) from Allameh Tabataba’i University, Iran. He earned his bachelor’s degree in IE (2007). His research interests include decision making under uncertainty and strategic management.

Mohammad Mahdi Mozaffari is an assistant professor of management in Imam Khomeini International University, Gazvin, Iran. He received his PhD from University of Tehran. His research interests include OR and Multi Attribute Decision Making.