Determination of Optimum Rails Dimensions in Railgun by Lagrange's Equations

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One of the methods that are used for increasing accelerator force of projectile in the railgun is the increasing of inductance gradient. Essentially, the inductance gradient is determined by current density in rails and projectile. If we change geometry of the rail, the current density and inductance gradient will be changed. One of the factors that restricts variation domain of rails geometry is the tolerable current of rails. In this paper, we have obtained analytical formulas for the maximum current density and the inductance gradient in terms of rail dimensions using the results that have been obtained by 2-D finite-element method. By these formulas and Lagrange's optimization equations, we determined the optimum dimensions of rail. Tolerable current is bonded for Lagrange's equations.

***Index Terms*—Lagrange's equations, optimization, railgun, 2-D finite-element method (FEM).**

I. Introduction

INDUCTANCE gradient is one of the important parameters to study the railgun performance. This parameter is directly proportional to the accelerator force [1]

$$F_{\text{Proj}} = \frac{1}{2} L' I^2 \tag{1}$$

where F is the accelerator force, I is the rail current, and L' is the inductance gradient. Inductance gradient depends on the following variables: the waveform of current [2], geometry of rails [3], and materials of rails [4] and projectile. Without attention to the practical limitations, we can increase the inductance gradient by decreasing the cross section of the rails and by increasing the space between them [5]. Practically, there are restrictions that do not allow us to increase the inductance gradient without limitations. One of the restrictions is the allowable current of rails. If the value of current is more than the allowable current, the rails will be melted. By decreasing the rail dimensions, the level of allowable current of rail is decreased.

In this paper, we present a method for the determination of optimum dimension of rails for the maximum value of gradient inductance, as current density in all points of the rail is not more than the allowable current density. For this purpose, we have obtained analytical formulas for the inductance gradient and the maximum current density. These formulas have been achieved from the results that were computed by finite-element method (FEM). After that, by Lagrange's equations and their bond, the optimum dimension of rail has been computed and was compared with the result of simulation (see Fig. 1).

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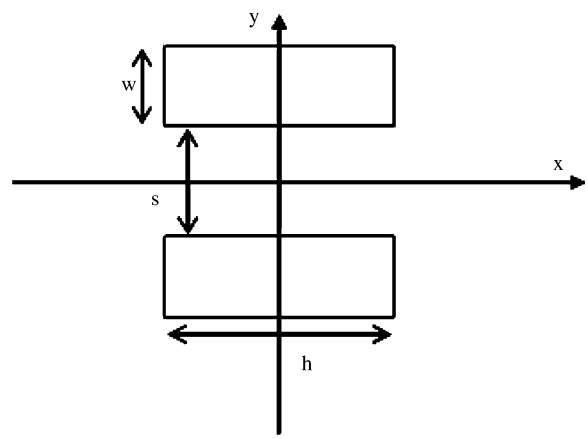

Fig. 1. Cross section of rails.

II. Problem Expression and Governing Equations

Lagrange's method is one of the famous methods for optimization [6]. In this method, primarily, we determine the goal function. This function depends on several variations. For optimization, the goal function must be minimum. Goal function has been shown with G as follows:

$$G(x_1, x_2, \ldots, x_n). \tag{2}$$

In addition to minimize the G, we want to follow the conditions as it is gratified

$$g_i(x_1, x_2, \ldots, x_n), \qquad i = 1, 2, \ldots, m. \tag{3}$$

In this method, Lagrange function is defined as follows:

$$P = G + \sum_{i=1}^{m} \lambda_i g_i. \tag{4}$$

TABLE I
INDUCTANCE GRADIENT AND THE MAXIMUM CURRENT DENSITY IN TERMS OF RAIL DIMENSIONS ($s = 2$ cm)

h(cm)	w = 1cm		w = 1.5cm		w = 2cm	
	L'	J_{max}	L'	J_{max}	L'	J_{max}
0.8	0.65757	54.480	0.64606	49.078	0.61017	45.673
1	0.65307	50.313	0.60882	45.637	0.57707	42.723
1.2	0.61460	46.977	0.57590	42.924	0.54804	40.316
1.4	0.58094	44.184	0.54681	40.601	0.52198	38.280
1.5	0.56570	42.947	0.53351	39.552	0.50993	37.355
1.6	0.55121	41.774	0.52079	38.566	0.49850	36.482
1.8	0.52453	39.662	0.49731	36.774	0.47722	34.881
2	0.50059	37.798	0.47601	35.164	0.45779	33.428
2.2	0.47886	36.119	0.45654	33.695	0.44000	32.117
2.5	0.44974	33.875	0.43037	31.747	0.41584	30.341

It has been proven that the point of inflection of P corresponds to the minimum of G.

$$\frac{\partial P(x_1, x_2, \ldots, x_n, \lambda_1, \lambda_2, \ldots, \lambda_n)}{\partial x_i} = 0, \qquad i = 1, 2, \ldots, n. \tag{5}$$

We obtain n equations from (5). With m equations in (3), there are $m + n$ linear equations. We have to multiply G in the minus mark to obtain the maximum of G.

In this paper, our goal function is the inductance gradient. Generally, for rectangular rails, L' is as follows:

$$L'\left(h, w, s, \rho, i(t)\right) \tag{6}$$

where h is the width of rail, w is the thickness of rail, s is the distance between rails, ρ is special resist of rail, and $i(t)$ is the electrical current. Whereas, it is possible to use from the harmonic analysis instead of transient analysis; also, we can replace $i(t)$ with frequency (f).

$$L'(h, w, s, \rho, f). \tag{7}$$

In this paper, we have used rails that are made of copper. Distance between rails (s) is 2 cm, and the frequency of current is constant. Therefore, L' is the function of width and thinness of rails $(L'(h, w))$. It is assumed that the tolerable current density of copper can be 40 $\mathrm{KA/mm^2}$.

Then

$$\begin{aligned} J_{\max} &\leq 40\,\mathrm{KA/mm^2} \\ J_{\max} &\leq 40\,\mathrm{GA/m^2}. \end{aligned} \tag{8}$$

Mostly, the maximum current density is concentrated in inner corners of rails. Maximum current density depends on variables of goal function. These variables are obtained from the simulation. There is one bond for this problem, and the inductance gradient is proportional to the current density. Then, we can ignore from the inequality in (8) and express the new bond as follows:

$$g(h, w) = J_{\max}(h, w) - 4 \times 10^{10}. \tag{9}$$

It is necessary to obtain analytical phrase for $J_{\max}$ in terms of h and w.

III. ANALYTICAL PHRASES FOR L', $J_{\max}$

In this section, we want to find analytical phrases for L' and $J_{\max}$ using the simulation results. For this purpose, we perform simulation using FEM. In simulation, the amplitude of ac current of rails is 0.6 MA. Values of L' and $J_{\max}$ are given in Table I.

We assume that L' can be a multiplication of two functions. Each of them is the function of independent variable. In other words, it is possible to separate variables

$$L'(h, w) = f(h)k(w). \tag{10}$$

After that, we test results for accuracy of this assumption. By using the results of Table I, each of functions $(f(h), k(w))$ is fitted with second order of polynomial function. This is shown in Fig. 2.

Equation (10) can be written as

$$L'(h, w) = f(h)k(w) = \left(\alpha_1 h^2 + \alpha_2 h + \alpha_3\right)\left(\beta_1 w^2 + \beta_2 w + \beta_3\right). \tag{11}$$

We assume that $\alpha_1\beta_1 = A_1$, $\alpha_1\beta_2 = A_2$. By recent assumption, the earlier equation can be rewritten as

$$\begin{aligned} L' = (A_1 w^2 + A_2 w + A_3)h^2 + (A_4 w^2 + A_5 w + A_6)h \\ +(A_7 w^2 + A_8 w + A_9). \end{aligned} \tag{12}$$

By fitting results of Table I

$$L'|_{w=1} = 0.04094h^2 - 0.27834h + 0.89166$$

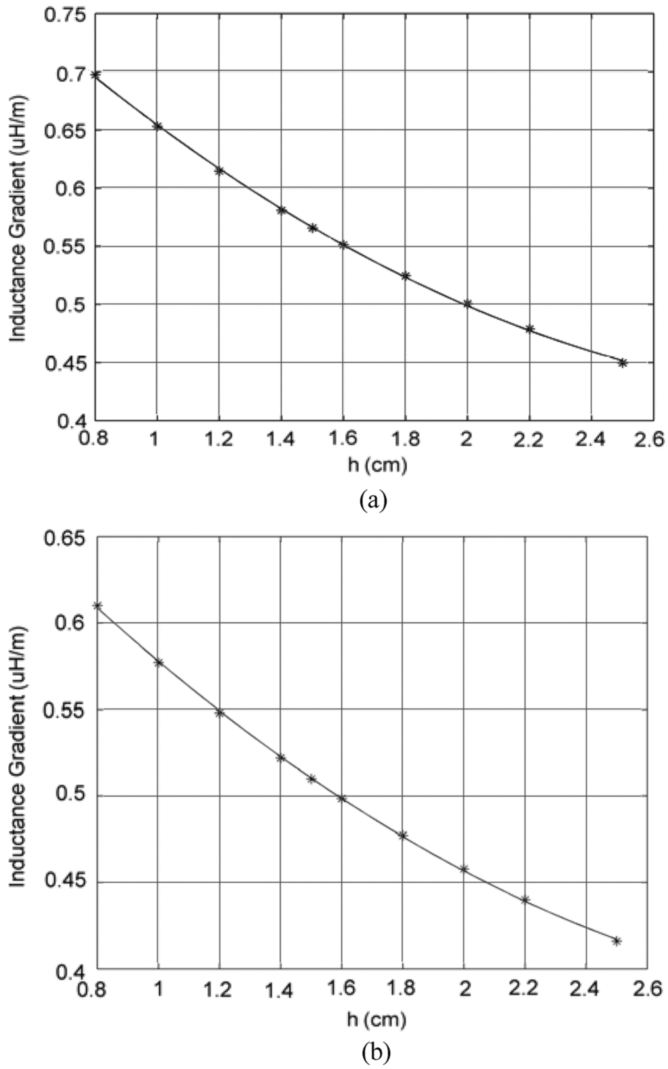

Fig. 2. Comparison of curve fitting and simulation results. (a) $w = 1$ cm. (b) $w = 2$ cm (simulation: $*$, Fit: —).

$$L'|_{w=1.5} = 0.03274h^2 - 0.23314h + 0.80996$$
$$L'|_{w=2} = 0.02782h^2 - 0.20461h + 0.75462. \quad (13)$$

If the coefficients of these equations is equal to the coefficients of (12), we have a system of linear equations with nine equations. By solving these equations, A_1 and A_2 will be obtained. In (12), by the determination of A_1 and A_2, the analytical phrase has been determined for L'. The accuracy of this phrase can be tested in comparison with the simulation of results. Fig. 3 shows it for $s = h = 2$ cm and the variation of w. The maximum error is less than 0.7%.

There is similar phrase for $J_{\max}$

$$J_{\max} = (B_1w^2 + B_2w + B_3)h^2 + (B_4w^2 + B_5w + B_6)h + (B_7w^2 + B_8w + B_9). \quad (14)$$

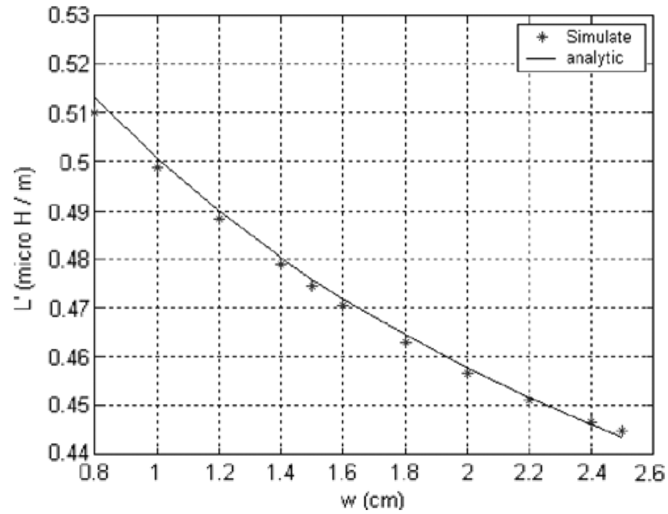

Fig. 3. Comparison of analytical phrases and simulation for $h = 2$ cm. Simulation: $*$. Analytic: —.

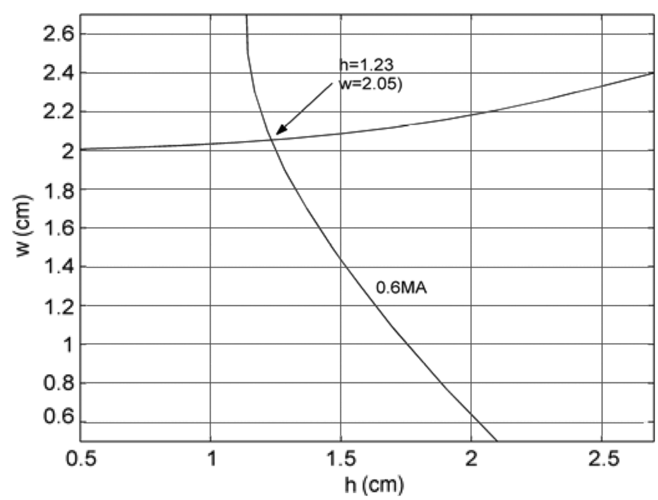

Fig. 4. Determination of the optimum dimension of rail in 0.6 MA.

IV. Optimum Dimensions of Rails

By using the analytical phrases, we write Lagrange's equation as follows:

$$P = -L'(h,w) + \lambda(J_{\max}(h,w) - 4 \times 10^{10}) = 0 \quad (15)$$

$$\frac{\partial P}{\partial h} = \frac{\partial}{\partial h}\left[-L'(h,w)\right] + \frac{\partial}{\partial h}\left[\lambda\left(J_{\max}(h,w) - 4 \times 10^{10}\right)\right] = 0 \quad (16)$$

$$\frac{\partial P}{\partial w} = \frac{\partial}{\partial w}\left[-L'(h,w)\right] + \frac{\partial}{\partial w}\left[\lambda\left(J_{\max}(h,w) - 4 \times 10^{10}\right)\right] = 0. \quad (17)$$

λ can be obtained from (15) in terms of L' and $J_{\max}$ and replaced in (16) and (17). Equations (16) and (17) formed a system of linear equations with two variables (L' and $J_{\max}$).

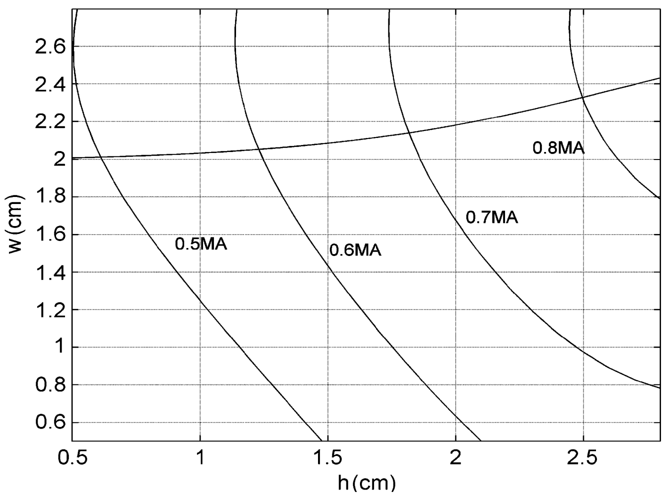

Fig. 5. Optimum dimension of rail for 0.5, 0.6, 0.7, and 0.8 MA.

For solving these equations, we have drowned two equations in one system of coordinates. Junction point of these curves is the answer of the problem.

For the input current equal to 0.6 MA, as shown in Fig. 4, the junction point is $w = 2.05\ \mathrm{cm}$, $h = 1.25$. In other words, for recent current, the rail with these dimensions has the maximum inductance gradient.

The accuracy of recently optimized dimensions has been verified with simulation. Results showed that the inductance gradient is equal to $0.54156\ \mu\mathrm{H/m}$, and the maximum current density is equal to $3.97902 \times 10^{10}\mathrm{A/m^2}$.

The obtained results in Fig. 5 shows that if the current is increased, the dimension of rails will be increased (for various current).

V. CONCLUSION

In this paper, we present a method in order to determine the optimum dimensions of rails for railgun. This method is based on the Lagrange optimization method. Bond of problem is the tolerable current. We used the results of FEM to obtain the analytical phrase for the inductance gradient and the maximum current density. We calculated the optimum dimensions of rails using these analytical phrases and the Lagrange equations. Results of the optimization confirm the results of simulation, and finally, results of this paper can be used for rail designing.

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